

Heisenberg XYZ Hamiltonian with Integrable Impurities

Zhan-Ning Hu¹

Institute of Physics and Center for Condensed Matter Physics,
Chinese Academy of Sciences, Beijing 100080, China

Abstract

In this letter, a Hamiltonian of the impurity model is constructed within the framework of the open boundary Heisenberg XYZ spin chain. This impurity model is an exactly solved one and it degenerates to the integrable XXZ impurity model under the triangular limit. This approach is the first time to add the integrable impurities to the completely anisotropic Heisenberg spin model with the open boundary conditions.

As quantum-mechanical model of magnetism, Heisenberg's model[1] is very fruitful in the theory of magnetism and is actively studied after Bethe's work on isotropic case of the XXX model. Yang and Yang[2] generalized Bethe's method to the XXZ model. Baxter[3] in his remarkable papers gave a solution for the completely anisotropic XYZ model with the use of the Bethe ansatz method. Faddeev and Takhtajan proposed the quantum inverse scattering method for the XYZ spin model and simplified Baxter's formulae[4]. Then, many exactly solved models have been proposed and solved by the coordinate Bethe ansatz[5], the functional Bethe ansatz[6, 7] and algebraic Bethe ansatz method [8, 9], etc. Recently the greatest progress has been made for the quantum impurity problems such as Kondo problem and tunneling in quantum wires for the one dimensional electron systems. We know that the impurities play an important role in the strongly correlated electron systems and even a small amount of defects may change the properties of the electron systems. Then it is very important to construct the integrable systems including the impurities. The pioneering work on the impurity model with the integrability was carried out by Andrei and Johannesson[10] for the isotropic Heisenberg chain. It were extended to the Babujian-Takhtajan spin chain in Refs. [11]. Bedüfig, Eßler and Frahm[12] solved the integrable model with the impurity coupled with periodic $t - J$ chain[13, 14, 15]. Schlottmann and Zvyagin have introduced the impurity in supersymmetric $t - J$ model via its scattering matrix with the itinerant electrons[16, 17]. The Hamiltonian of the system and other conserved currents can be constructed in principle by the transfer matrix. They have discussed also the magnetic impurities embedded in the Hubbard model[18] and a finite concentration of magnetic impurities embedded in one-dimensional lattice via scattering matrices[19].

Exactly solved systems with the open boundary conditions have been studied earlier in Refs. [20-24] and a general approach to construct open quantum spin model is given by Cherednik[25] and Sklyanin[26]. Then many exactly solved model

¹E-mail: huzn@aphy.iphy.ac.cn

with the boundary conditions have been proposed and solved after these pioneering work [27-42]. We notice that the impurities may cut the one-dimensional system into the small part when they are introduced and (then) the open boundary systems are formed with the impurities at the ends of the systems. Then the integrable impurity model [43-49] can be constructed from the open boundary system. In this paper, we devote to construct an integrable Hamiltonian of the impurity model for the completely anisotropic XYZ Heisenberg spin chain where the impurities are coupled to the ends of the system. As is well known, the XYZ spin model is described by the Hamiltonian

$$H = \sum_{n=1}^{N-1} \left(J_1 \sigma_n^1 \sigma_{n+1}^1 + J_2 \sigma_n^2 \sigma_{n+1}^2 + J_3 \sigma_n^3 \sigma_{n+1}^3 \right) \quad (1)$$

where J_1 , J_2 and J_3 are constants; the spin operators σ_n^j have the form

$$\sigma_n^j = I \otimes \cdots \otimes \sigma^j \otimes \cdots \otimes I \quad (j = 1, 2, 3; n = 1, \dots, N)$$

where σ^j at site n are the Pauli operators. This Hamiltonian denotes a one-dimensional quantum mechanical model of the ferromagnetism. There are N spins labelled by $n = 1, 2, \dots, N$ on a line. Every spin is associated with the three-dimensional vector $\vec{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ of the Pauli matrices. The interactions between the neighboring spins are expressed by the constants $J_{1,2,3}$. The R matrix of the system has the form

$$R(\lambda) = \begin{pmatrix} \text{sn}(\lambda + \eta) & 0 & 0 & k \text{sn} \eta \text{sn} \lambda \text{sn}(\lambda + \eta) \\ 0 & \text{sn} \lambda & \text{sn} \eta & 0 \\ 0 & \text{sn} \eta & \text{sn} \lambda & 0 \\ k \text{sn} \eta \text{sn} \lambda \text{sn}(\lambda + \eta) & 0 & 0 & \text{sn}(\lambda + \eta) \end{pmatrix}, \quad (2)$$

which satisfies the Yang-Baxter equation

$$\begin{aligned} & R_{12}(\lambda_1 - \lambda_2) R_{13}(\lambda_1 - \lambda_3) R_{23}(\lambda_2 - \lambda_3) \\ & = R_{23}(\lambda_2 - \lambda_3) R_{13}(\lambda_1 - \lambda_3) R_{12}(\lambda_1 - \lambda_2) \end{aligned} \quad (3)$$

and has the properties

$$\begin{aligned} & R_{12}(\lambda) R_{12}(-\lambda) = \rho(\lambda) I, \\ & R_{12}(0) = \text{sn} \eta P_{12}, R_{12}^{t_1 t_2}(\lambda) = R_{12}(\lambda), \\ & R_{12}^{t_1}(\lambda) R_{12}^{t_1}(-\lambda - \eta) = \rho(\lambda + \eta) I \end{aligned} \quad (4)$$

with $\rho(\lambda) = \text{sn}^2 \eta - \text{sn}^2 \lambda$. Another kind of the basic properties in the above Yang-Baxter equation is the difference property that the R matrices rely on only the differences of the corresponding spectrum parameters, which is useful to construct

the models with integrable impurities. The boundary K matrices satisfy the reflection equation[25, 26]

$$\begin{aligned} & R_{12}(\lambda_1 - \lambda_2) \overset{1}{K}_-(\lambda_1) R_{12}(\lambda_1 + \lambda_2) \overset{2}{K}_-(\lambda_2) \\ = & \overset{2}{K}_-(\lambda_2) R_{12}(\lambda_1 + \lambda_2) \overset{1}{K}_-(\lambda_1) R_{12}(\lambda_1 - \lambda_2) \end{aligned} \quad (5)$$

$$\begin{aligned} & R_{12}(-\lambda_1 + \lambda_2) \overset{1}{K}_+(\lambda_1) R_{12}(-\lambda_1 - \lambda_2 - 2\eta) \overset{2}{K}_+(\lambda_2) \\ = & \overset{2}{K}_+(\lambda_2) R_{12}(-\lambda_1 - \lambda_2 - 2\eta) \overset{1}{K}_+(\lambda_1) R_{12}(-\lambda_1 + \lambda_2) \end{aligned} \quad (6)$$

where $\overset{1}{K}_\pm \equiv K_\pm \otimes id_{V_2}$ and $\overset{2}{K}_\pm \equiv id_{V_1} \otimes K_\pm$. Here I follow the same notations as in Refs. [26-29]. The transfer matrix $t(\lambda)$ is defined as

$$t(\lambda) = tr \left\{ K_+(\lambda) T(\lambda) K_-(\lambda) T^{-1}(-\lambda) \right\} \quad (7)$$

with the use of the boundary K_\pm matrices and the monodromy matrix $T(\lambda)$. It has the commuting property

$$[t(\lambda), t(\lambda')] = 0,$$

which can be obtained from the Yang-Baxter equation and the boundary reflection equations (5) and (6). This property ensures the integrability of the system with the open boundary conditions. For convenience, we use the notation

$$R(\lambda) = \sum_{j=1}^4 W_j(\lambda) \sigma^j \otimes \sigma^j \quad (8)$$

where

$$\begin{aligned} W_4(\lambda) + W_3(\lambda) &= \text{sn}(\lambda + \eta), \\ W_4(\lambda) - W_3(\lambda) &= \text{sn}\lambda, \\ W_1(\lambda) + W_2(\lambda) &= \text{sn}\eta, \\ W_1(\lambda) - W_2(\lambda) &= k \text{sn}\eta \text{sn}\lambda \text{sn}(\lambda + \eta). \end{aligned} \quad (9)$$

In order to add the magnetic impurities to the XYZ spin chain, we define that

$$T_a(\lambda) = R_{aR}(\lambda + c_R) R_{aN}(\lambda) R_{aN-1}(\lambda) \cdots R_{a1}(\lambda). \quad (10)$$

Then the inverse of the monodromy matrix is

$$T_a^{-1}(-\lambda) = \frac{1}{\rho^N(\lambda) \rho(\lambda - c_R)} R_{a1}(\lambda) R_{a2}(\lambda) \cdots R_{aN}(\lambda) R_{aR}(\lambda - c_R).$$

We choose that the boundary K matrices as

$$\begin{aligned} K_-(\lambda) &= R_{aL}(\lambda + c_L) R_{aL}(\lambda - c_L), \\ K_+(\lambda) &= 1, \end{aligned} \quad (11)$$

which satisfy the reflection equations. Here the impurity parameters c_R and c_L are introduced. They are related to the exchange constants between the particles of the system and the impurities situated at the ends of the chain. These exchange couplings between the boundary spins can be turned away from their bulk values by variation of the parameters c_R and c_L respectively. It occurs since the construction is based completely on the difference properties of the solutions of the Yang-Baxter equation (3). The transfer matrix is

$$t(\lambda) = \text{Tr} \left\{ T_a(\lambda) K_-(\lambda) T_a^{-1}(-\lambda) \right\}.$$

Then we have that

$$\begin{aligned} & \text{Tr}_a \left\{ \left. \frac{dR_{aR}(\lambda + c_R)}{d\lambda} \right|_{\lambda=0} R_{aN}(0) R_{aN-1}(0) \cdots R_{a1}(0) K_-(0) T_a^{-1}(0) \right\} \\ &= \frac{2\rho(c_L)}{\rho(c_R)} \sum_{j=1}^4 \left. \frac{dW_j(\lambda + c_R)}{d\lambda} \right|_{\lambda=0} W_j(-c_R), \end{aligned} \quad (12)$$

$$\begin{aligned} & \text{Tr}_a \left\{ R_{aR}(c_R) \left. \frac{dR_{aN}(\lambda)}{d\lambda} \right|_{\lambda=0} R_{aN-1}(0) \cdots R_{a1}(0) K_-(0) T_a^{-1}(0) \right\} \\ &= \frac{\rho(c_L)}{\text{sn}\eta\rho(c_R)} \left\{ 2J_1 A_R \sigma_N^1 \sigma_R^1 + 2J_2 B_R \sigma_N^2 \sigma_R^2 + 2J_3 C_R \sigma_N^3 \sigma_R^3 \right. \\ & \quad \left. + 2J_3 \sum_{j=1}^4 W_j(c_R) W_j(-c_R) \right\}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} A_R &= W_1(c_R) W_4(-c_R) + W_4(c_R) W_1(-c_R) \\ & \quad + W_2(c_R) W_3(-c_R) + W_3(c_R) W_2(-c_R), \end{aligned} \quad (14)$$

$$\begin{aligned} B_R &= W_1(c_R) W_3(-c_R) + W_3(c_R) W_1(-c_R) \\ & \quad + W_2(c_R) W_4(-c_R) + W_4(c_R) W_2(-c_R), \end{aligned} \quad (15)$$

$$\begin{aligned} C_R &= W_1(c_R) W_2(-c_R) + W_2(c_R) W_1(-c_R) \\ & \quad + W_3(c_R) W_4(-c_R) + W_4(c_R) W_3(-c_R). \end{aligned} \quad (16)$$

By the use of

$$\begin{aligned}
P_{nn+1} \frac{dR_{nn+1}(\lambda)}{d\lambda} \Big|_{\lambda=0} &= \frac{1 + k \operatorname{sn}^2 \eta}{2} \sigma_n^1 \sigma_{n+1}^1 + \frac{1 - k \operatorname{sn}^2 \eta}{2} \sigma_n^2 \sigma_{n+1}^2 \\
&\quad + \frac{\operatorname{cn} \eta \operatorname{dn} \eta}{2} \sigma_n^3 \sigma_{n+1}^3 + \frac{\operatorname{cn} \eta \operatorname{dn} \eta}{2},
\end{aligned} \tag{17}$$

we have that

$$\begin{aligned}
&\sum_{j=1}^{N-1} \operatorname{Tr}_a \left\{ R_{aR}(c_R) R_{aN}(0) \cdots R_{aj+1}(0) \frac{dR_{aj}(\lambda)}{d\lambda} \Big|_{\lambda=0} \right. \\
&\quad \left. \cdot R_{aj-1}(0) \cdots R_{a1}(0) K_-(0) T_a^{-1}(0) \right\} \\
&= \frac{2\rho(c_L)}{\operatorname{sn} \eta} \sum_{j=1}^{N-1} \left(J_1 \sigma_n^1 \sigma_{n+1}^1 + J_2 \sigma_n^2 \sigma_{n+1}^2 + J_3 \sigma_n^3 \sigma_{n+1}^3 + J_3 \right)
\end{aligned} \tag{18}$$

where $J_1 = (1 + k \operatorname{sn}^2 \eta)/2$, $J_2 = (1 - k \operatorname{sn}^2 \eta)/2$ and $J_3 = \operatorname{cn} \eta \operatorname{dn} \eta/2$; and

$$\begin{aligned}
&\frac{1}{\rho^N(\lambda) \rho(\lambda - c_R)} \operatorname{Tr}_a \left\{ T_a(0) K_-(0) R_{a1}(0) \cdots \right. \\
&\quad \left. \cdot R_{aN}(0) \frac{dR_{aR}(\lambda + c_R)}{d\lambda} \Big|_{\lambda=0} \right\} \\
&= \frac{2\rho(c_L)}{\rho(c_R)} \sum_{j=1}^4 W_j(c_R) \frac{dW_j(\lambda - c_R)}{d\lambda} \Big|_{\lambda=0},
\end{aligned} \tag{19}$$

$$\begin{aligned}
&\operatorname{Tr}_a \left\{ T_a(0) \frac{dK_-(\lambda)}{d\lambda} \Big|_{\lambda=0} T_a^{-1}(0) \right\} \\
&= 2A_L \sigma_1^1 \sigma_L^1 + 2B_L \sigma_1^2 \sigma_L^2 + 2C_L \sigma_1^3 \sigma_L^3 + D_L
\end{aligned} \tag{20}$$

where

$$\begin{aligned}
A_L &= \frac{dW_1(\lambda + c_L)}{d\lambda} \Big|_{\lambda=0} W_4(-c_L) + \frac{dW_4(\lambda + c_L)}{d\lambda} \Big|_{\lambda=0} W_1(-c_L) \\
&\quad - \frac{dW_2(\lambda + c_L)}{d\lambda} \Big|_{\lambda=0} W_3(-c_L) - \frac{dW_3(\lambda + c_L)}{d\lambda} \Big|_{\lambda=0} W_2(-c_L) \\
&\quad + \frac{dW_1(\lambda - c_L)}{d\lambda} \Big|_{\lambda=0} W_4(c_L) + \frac{dW_4(\lambda - c_L)}{d\lambda} \Big|_{\lambda=0} W_1(c_L) \\
&\quad - \frac{dW_2(\lambda - c_L)}{d\lambda} \Big|_{\lambda=0} W_3(c_L) - \frac{dW_3(\lambda - c_L)}{d\lambda} \Big|_{\lambda=0} W_2(c_L),
\end{aligned} \tag{21}$$

$$\begin{aligned}
B_L = & \left. \frac{dW_2(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_4(-c_L) + \left. \frac{dW_4(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_2(-c_L) \\
& - \left. \frac{dW_1(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_3(-c_L) - \left. \frac{dW_3(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_1(-c_L) \\
& + \left. \frac{dW_2(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_4(c_L) + \left. \frac{dW_4(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_2(c_L) \\
& - \left. \frac{dW_1(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_3(c_L) - \left. \frac{dW_3(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_1(c_L), \quad (22)
\end{aligned}$$

$$\begin{aligned}
C_L = & \left. \frac{dW_3(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_4(-c_L) + \left. \frac{dW_4(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_3(-c_L) \\
& - \left. \frac{dW_1(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_2(-c_L) - \left. \frac{dW_2(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_1(-c_L) \\
& + \left. \frac{dW_3(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_4(c_L) + \left. \frac{dW_4(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_3(c_L) \\
& - \left. \frac{dW_1(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_2(c_L) - \left. \frac{dW_2(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} W_1(c_L), \quad (23)
\end{aligned}$$

$$D_L = \sum_{j=1}^4 \left\{ \left. \frac{dW_j(\lambda + c_L)}{d\lambda} \right|_{\lambda=0} W_j(-c_L) + W_j(c_L) \left. \frac{dW_j(\lambda - c_L)}{d\lambda} \right|_{\lambda=0} \right\}. \quad (24)$$

Therefore we get that

$$\begin{aligned}
\frac{\text{sn}\eta}{4\rho(c_L)} \left. \frac{dt(\lambda)}{d\lambda} \right|_{\lambda=0} = & \sum_{n=1}^{N-1} \left(J_1 \sigma_n^1 \sigma_{n+1}^1 + J_2 \sigma_n^2 \sigma_{n+1}^2 + J_3 \sigma_n^3 \sigma_{n+1}^3 \right) \\
& + \frac{\text{sn}\eta}{2\rho(c_L)} \left(A_L \sigma_1^1 \sigma_L^1 + B_L \sigma_1^2 \sigma_L^2 + C_L \sigma_1^3 \sigma_L^3 \right) \\
& + \frac{1}{\rho(c_R)} \left(J_1 A_R \sigma_N^1 \sigma_R^1 + J_2 B_R \sigma_N^2 \sigma_R^2 + J_3 C_R \sigma_N^3 \sigma_R^3 \right) \\
& + \frac{\text{sn}\eta}{2\rho(c_L)} C_L + \frac{\text{sn}\eta}{2\rho(c_R)} C_L + \frac{J_3}{\rho(c_R)} \sum_{j=1}^4 W_j(c_R) W_j(-c_R) \\
& + (N-1) J_3. \quad (25)
\end{aligned}$$

Set

$$H \equiv \frac{\text{sn}\eta}{4\text{sn}^2\eta - 4\text{sn}^2c_L} \left. \frac{dt(\lambda)}{d\lambda} \right|_{\lambda=0} - h_0 \quad (26)$$

with

$$h_0 = N J_3 + \frac{1}{4} \text{sn}\eta \text{sn}(2\eta) \left(1 - k^2 \text{sn}^2\eta \text{sn}^2c_L \right)$$

$$\begin{aligned}
& \cdot \left[1 - k^2 \text{sn}^2(\eta + c_L) \text{sn}^2(\eta - c_L) \right] \\
& \cdot \left(\frac{1}{\text{sn}^2 \eta - \text{sn}^2 c_L} + \frac{1}{\text{sn}^2 \eta - \text{sn}^2 c_R} \right). \tag{27}
\end{aligned}$$

We get the following Hamiltonian of the impurity model,

$$\begin{aligned}
H &= \sum_{n=1}^{N-1} \left(J_1 \sigma_n^1 \sigma_{n+1}^1 + J_2 \sigma_n^2 \sigma_{n+1}^2 + J_3 \sigma_n^3 \sigma_{n+1}^3 \right) \\
&+ \frac{1}{\text{sn}^2 \eta - \text{sn}^2 c_L} \left\{ J_{L,1} \sigma_1^1 \sigma_L^1 + J_{L,2} \sigma_1^2 \sigma_L^2 + J_{L,3} \sigma_1^3 \sigma_L^3 \right\} \\
&+ \frac{1}{\text{sn}^2 \eta - \text{sn}^2 c_R} \left\{ J_{R,1} \sigma_N^1 \sigma_R^1 + J_{R,2} \sigma_N^2 \sigma_R^2 + J_{R,3} \sigma_N^3 \sigma_R^3 \right\}, \tag{28}
\end{aligned}$$

where

$$\begin{aligned}
J_1 &= \frac{1 + k \text{sn}^2 \eta}{2}, \quad J_2 = \frac{1 - k \text{sn}^2 \eta}{2}, \quad J_3 = \frac{\text{cn} \eta \text{dn} \eta}{2}, \\
J_{L,1} &= \frac{1}{2} \text{sn}^2 \eta \text{cn} c_L \text{dn} c_L [1 + k \text{sn}(\eta + c_L) \text{sn}(\eta - c_L)], \\
J_{L,2} &= \frac{1}{2} \text{sn}^2 \eta \text{cn} c_L \text{dn} c_L [1 - k \text{sn}(\eta + c_L) \text{sn}(\eta - c_L)], \\
J_{L,3} &= \frac{1}{4} \text{sn} \eta \text{sn}(2\eta) (1 - k^2 \text{sn}^2 \eta \text{sn}^2 c_L) \\
&\cdot \left[1 - k^2 \text{sn}^2(\eta + c_L) \text{sn}^2(\eta - c_L) \right].
\end{aligned}$$

And the exchange constants $J_{R,i}$ have the same expressions as the constants $J_{L,i}$ ($i = 1, 2, 3$) except for the substitution of the parameters c_L by c_R . It is an integrable impurity model. The two impurities are coupled to both of the ends of the completely anisotropic Heisenberg spin chain. When we take the triangular limit $k \rightarrow 0$ (the supplementary modulus k' tends to 1), the elliptic functions $\text{sn} u, \text{cn} u, \text{dn} u$ become $\sin u, \cos u$, and 1, respectively. The above Hamiltonian reduces to

$$\begin{aligned}
H &= \frac{1}{2} \left\{ \sum_{n=1}^{N-1} \left(\sigma_n^1 \sigma_{n+1}^1 + \sigma_n^2 \sigma_{n+1}^2 + \cos \eta \sigma_n^3 \sigma_{n+1}^3 \right) \right. \\
&+ \frac{\sin^2 \eta \cos c_L}{\sin^2 \eta - \sin^2 c_L} \left(\sigma_1^1 \sigma_L^1 + \sigma_1^2 \sigma_L^2 + \frac{\cos \eta}{\cos c_L} \sigma_1^3 \sigma_L^3 \right) \\
&+ \left. \frac{\sin^2 \eta \cos c_R}{\sin^2 \eta - \sin^2 c_R} \left(\sigma_N^1 \sigma_R^1 + \sigma_N^2 \sigma_R^2 + \frac{\cos \eta}{\cos c_R} \sigma_N^3 \sigma_R^3 \right) \right\}. \tag{29}
\end{aligned}$$

It is just the Hamiltonian of the impurity model related to XXZ spin model[47, 50].

As the conclusion, we have added the magnetic impurities to the edges of the completely anisotropic Heisenberg XYZ spin model. The model is exactly solvable and the Hamiltonian of the impurity model is obtained explicitly. The interactions

between the impurities and the electrons are described by the arbitrary parameters c_L and c_R , which comes to the difference properties of the spectrum parameters of the R matrices of the spin chain. Under the triangular limit, our Hamiltonian of the impurity model reduces to the one related to the XXZ spin model[47]. We know that the XYZ spin model is equivalent to the eight vertex model[6] and some progress has been made for the eight vertex models with the open boundary conditions[51, 52]. Batchelor *et al*[40] have obtained the surface free energy with the boundary K matrix[53]. It is an interesting subject to discuss the impurity effects in the two dimensional lattice model in statistical mechanics. Furthermore, the Bethe ansatz equations of this system with the impurities can be obtained by the use of the general Bethe ansatz method. We may study the thermodynamics of the XYZ model including the magnetic impurities and the impurity effects in the ground state and excited state. All of these are in investigations.

References

- [1] W. Heisenberg, Z. Phys. 49 (1928) 619.
- [2] C.N. Yang and C.P. Yang, Phys. Rev. 150 (1966) 321; Phys. Rev. 150 (1966) 327; Phys. Rev. 111 (1966) 258.
- [3] R.J. Baxter, Phys. Rev. Lett. 26 (1971) 832; Phys. Rev. Lett. 26 (1971) 834; Ann.Phys. 80 (1972) 193; Ann.Phys. 80 (1972) 323.
- [4] L.A. Takhtajan and L.D. Faddeev, Russ. Math. Surv. 34 (1979) 11.
- [5] E. H. Lieb and F. Y. Wu, Phase Transitions and Critical Phenomena, ed. Domb C and Green M S (Academic Press, London) 1 (1982) 61.
- [6] R.J. Baxter, Exactly Solved Models in Statistical Mechanics (Academic Press, London, 1982).
- [7] V.V. Bazhanov and N.Yu. Reshetikhin, Int. J. Mod. Phys. B 4 (1989) 115.
- [8] O. Babelon, H. J. de Vega and C. M. Viallet, Nucl. Phys. B 200 (1982) 266.
- [9] C.L. Schultz, Physica A 122 (1983) 71.
- [10] N. Andrei and H. Johnesson, Phys. Lett. A 100 (1984) 108.
- [11] K. Lee and P. Schlottmann, Phys. Rev. B 37 (1988) 379; P. Schlottmann, J. Phys. Condens. Matter 3 (1991) 6617.
- [12] G. Bedürftig, F.H.L. Eßler and H. Frahm, Phys. Rev. Lett. 69 (1996) 5098; Nucl. Phys. B 489 (1997) 697.

- [13] P. Schlottmann, Phys. Rev. B 36 (1987) 5177.
- [14] P. -A. Bares, G. Blatter and M. Ogata, Phys. Rev. B 44 (1991) 130.
- [15] G. Jüttner, A. Klümper and J. Suzuki, Nucl. Phys. B 487 (1997) 650.
- [16] P. Schlottmann and A.A. Zvyagin, Phys. Rev. B 55 (1997) 5027; A. A. Zvyagin and P. Schlottmann, J. Phys. Conds. Matt. 9 3543; E, 9 (1997) 6479.
- [17] P. Schlottmann and A. A. Zvyagin, Nucl. Phys. B 501 (1997) 728; A. A. Zvyagin, Phys. Rev. Lett. 79 (1997) 4641.
- [18] A.A. Zvyagin and P. Schlottmann, Phys. Rev. B 56 (1997) 300.
- [19] P. Schlottmann and A. A. Zvyagin, Phys. Rev. B 56 (1997) 13989.
- [20] B.M. McCoy and T.T. Wu, Phys. Rev. 162 (1967) 436; Phys. Rev. 174 (1968) 546.
- [21] M. Gaudin, Phys. Rev. A 4 (1971) 386.
- [22] R.Z. Bariev, Theor Math Phys 40 (1979) 623.
- [23] J.L. Cardy, Nucl. Phys. B 275 (1986) 200.
- [24] F.C. Alcaraz, M.N. Barber, M.T. Batchelor, R.J. Baxter and G.R.W. Quispel, J. Phys. A 20 (1987) 6397.
- [25] I.V. Cherednik, Theor. Math. Phys. 17 (1983) 77; Theor. Math. Phys 61 (1984) 911.
- [26] E.K. Sklyanin, J. Phys. A 21 (1988) 2375.
- [27] C. Destri and H.J. de Vega, Nucl. Phys. B 374 (1992) 692; Nucl. Phys. B 385 (1992) 361.
- [28] L. Mezincescu and R.I. Nepomechie, J. Phys. A 24 (1991) L17; Mod. Phys. Lett. A 6 (1991) 2497; Nucl. Phys. B 372 (1992) 597.
- [29] P.P. Kulish and E.K. Sklyanin, J. Phys. A 24 (1991) L435.
- [30] A. Foerster and M. Karowski, Nucl. Phys. B 408 (1993) 512.
- [31] H. de Vega and A. Gonzalez-Ruiz, Nucl. Phys. B 417 (1994) 553; Mod. Phys. Lett. A 9 (1994) 2207.
- [32] R. Yue, H. Fan and B. Hou, Nucl. Phys. B 462 (1996) 167.
- [33] P. Fendley, S. Saleur and N.P. Warner, Nucl. Phys. B 430 (1995) 577; Nucl. Phys. B 428 (1994) 681.
- [34] S. Ghoshal and A.B. Zamolodchikov, Int. J. Mod. Phys. A 9 (1994) 3841.

- [35] M. Jimbo, K. Kedem, T. Kojima, H. Konno and T. Miwa, Nucl. Phys. B 441 (1995) 437.
- [36] C.M. Yung and M.T. Batchelor, Nucl. Phys. B 435 (1995) 430.
- [37] Y.K. Zhou, Nucl. Phys. B 453 (1995) 619.
- [38] T. Takebe, J. Phys. A 25 (1992) 1071; H. Fan, B.Y. Hou and K.J. Shi, J. Phys. A 28 (1995) 4743.
- [39] C. Ahn and W.M. Koo, hep-th/9508080.
- [40] M.T. Batchelor and Y.K. Zhou, Phys. Rev. Lett. 76 (1996) 2826.
- [41] T. Inami, S. Odake and Y.Z. Zhang, Phys. Lett. B 359 (1995) 118.
- [42] H.J. de Vega, Int. J. Mod. Phys. A 4 (1989) 2371.
- [43] Y. Wang and J. Voit, Phys. Rev. Lett. 77 (1996) 4934.
- [44] Y. Wang, J. Dai, Z.N. Hu and F.C. Pu, Phys. Rev. Lett. 79 (1997) 1901.
- [45] Z.N. Hu, F.C. Pu and Y. Wang, J. Phys. A 31 (1998) 5241.
- [46] Z.N. Hu and F.C. Pu, Effects of Magnetic Impurities in the $t-J$ Model, Preprint (1998).
- [47] Z.N. Hu and F.C. Pu, Phys. Rev. B 58 (1998) R2925.
- [48] Y. Wang, Phys. Rev. B 56 (1997) 14045.
- [49] Z.N. Hu and F.C. Pu, Two Magnetic Impurities with Arbitrary Spins in Open Boundary $t-J$ Model, Preprint, May (1998).
- [50] S. Chen, Y. P. Wang and F.C. Pu, J. Phys. A 31 (1998) 4619.
- [51] H. Fan, B.Y. Hou, K.J. Shi and Z.X. Yang, Nucl. Phys. B 478 (1996) 723.
- [52] M. Jimbo, K. Kedem, T. Kojima, H. Konno and T. Miwa, Nucl. Phys. B 448 (1995) 429.
- [53] Y.K. Zhou, Nucl. Phys. B 458 (1996) 504.